Interval Estimation Naïve Bayes

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Abstract. Recent work in supervised learning has shown that a surprisingly simple Bayesian classifier with assumptions of conditional independence among features given the class, called naïve Bayes, is competitive with state of the art classifiers. On this paper a new naïve Bayes classifier called *Interval Estimation naïve Bayes* is proposed. Interval Estimation naïve Bayes performs on two phases. On the first phase an interval estimation of each probability necessary to specify the naïve Bayes is estimated. On the second phase the best combination of values inside these intervals is calculated with a heuristic search that is guided by the accuracy of the classifiers. The founded values in the search are the new parameters for the naïve Bayes classifier. Our new approach has shown to be quite competitive related to simple naïve Bayes. Experimental tests have been done with 21 data sets from the UCI repository.

1 Introduction

The naïve Bayes classifier [3, 8] is a probabilistic method for classification. It can be used to determine the probability that an example belongs to a class given the values of attributes. The naïve Bayes classifier guarantees optimal induction given a set of explicit assumptions [1]. However, it is known that some of these assumptions are not compliant in many induction scenarios, for instance, the condition of variable independence respecting to the class variable. Improvements of accuracy has been demonstrated by a number of approaches, collectively named semi naïve Bayes classifiers, which try to adjust the naïve Bayes to deal with a-priori unattended assumptions.

Previous semi naïve Bayes classifiers can be divided into three groups, depending on different pre-/post-processing issues: (i) to manipulate the variables to be employed prior to application of naïve Bayes induction [12, 14, 18], (ii) to select subsets of the training examples prior to the application of naïve Bayes classification [10, 13] and (iii) to correct the probabilities produced by the standard naïve Bayes [21, 24].

We propose a new semi naïve Bayes approach named *Interval Estimation naïve Bayes (IENB)*, that tries to break the assumption of independence of the variables given the class by searching for the best combination of naïve Bayes probabilities inside its confidence intervals. There are some related work on this approximation. In [6] it is described an algorithm named *Iterative Bayes*, that tries to improve the conditional probabilities of naïve Bayes in an iterative way with a hill-climbing approach. In [19] is presented an algorithm named *Robust Bayes Classifier (RBC)*, a naïve Bayes classifier

designed for the case in which learning instances are incomplete. RBC takes into account all the possibilities for the missing values, calculating an interval for each naïve Bayes probability. In [22, 23] is presented a naïve Bayes extension based on credal approximation *naïve Credal Classifier (NCC)*. Credal sets represent probability distributions as points that belong to a close and fenced geometrical regions.

Interval Estimation naïve Bayes has been implemented in Visual C++ and the experimental evaluation has been done with 21 problems from the UCI database [17].

The outline of this paper is as follows: Section 2 presents the naïve Bayes classifier. Section 3 is a brief introduction to statistical inference. Section 4 presents the new algorithm Interval Estimation naïve Bayes. Section 5 illustrates the results with the UCI experiments. Section 6 gives the conclusions and suggests further future work.

2 Naïve Bayes

The naïve Bayes classifier [3, 8] is a probabilistic method for classification. It performs and approximate calculation of the probability that an example belongs to a class given the values of variables. The simple naïve Bayes classifier is one of the most successful algorithms on many classification domains. In spite of its simplicity, it is shown to be competitive with other more complex approaches in several specific domains.

This classifier learns from training data the conditional probability of each variable X_k given the class label c . Classification is then done by applying Bayes rule to compute the probability of C given the particular instance of $X_1, \dots, X_n$,

$$P(C = c | X_1 = x_1, \dots, X_n = x_n)$$

Naïve Bayes is founded on the assumption than variables are independent. Therefore this probability (given the value of the class) are formulated as follows,

$$P(C = c | X_1 = x_1, \dots, X_n = x_n) \propto P(C = c) \prod_{k=1}^n P(X_k = x_k | C = c) \quad (1)$$

This equation is highly appropriate for learning from data, since the probabilities $p_i = P(C = c^i)$ and $p_{k,r}^i = P(X_k = x_k^r | C = c^i)$ may be estimated from training data. The result of the classification is the class with highest probability.

In naïve Bayes these parameters are estimated using a point estimation (see section xxx). The first step in the new algorithm we propose is based on an interval estimation (see section xxx).

3 Parameter estimation

Statistical inference studies a collection of data based on a sample of these ones. This sample represents the part of population considered in the analysis. Amongst other things, statistical inference studies the problem known as “estimation problem”.

There are two ways of accomplishing this task:

- *Point Estimation*: Point estimation uses a sample with the aim of assigning a single value to a parameter. Maximum likelihood method and method of moments are used in this context.
- *Interval Estimation*: This technique calculates for each sample an interval that probably contains the parameter. This interval is called confidence interval. The probability of a parameter to be included in an interval is known as confidence level.

3.1 Point estimation of parameters in naïve Bayes

Considering the instances of the database $\mathcal{D} = \{(x_1, c_1), \dots, (x_N, c_N)\}$ as a random sample on size N where the predictor variables $X_1, \dots, X_N$ follow Bernoulli distributions and using the maximum likelihood method, the next intuitive results are reached:

$$\hat{P}_{k,r}^i = \frac{N_{kri}}{N_i} \quad (2)$$

where,

N_{kri} denotes the number of instances where $X_k = x_k^r$ and $C = c^i$

N_i denotes the number of instances where $C = c^i$

$$\hat{P}_i = \frac{N_i}{N} \quad (3)$$

being N is the size of the database.

3.2 Interval estimation of parameters in IENB

In the case of IENB what we need is to calculate the confidence intervals of the parameters. This estimation is achieved by the calculation of the sum of the variables in the sample, which generate a binomial distribution that can be approximated by a normal distribution, given the next result,

$$p \in \left(\hat{p} - z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{M}}; \hat{p} + z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{M}} \right) \quad (4)$$

where,

$\hat{p}$ is the point estimation of the probability

z_α is the $(1 - \frac{\alpha}{2})$ percentil in the $\mathcal{N}(0,1)$

M is the corresponding sample size (N_i or N)

4 Interval Estimation Naïve Bayes – IENB

We propose a new semi naïve Bayes approach named interval estimation naïve Bayes (IENB). In this approach, instead of calculating the point estimation of the conditional probabilities from data, as simple naïve Bayes does, confidence intervals are calculated. After that, by searching for the best combination of values into these intervals, we seek to break the assumption of independence among variables the simple naïve Bayes does.

This search is carry out by a heuristic search algorithm and is guided by the accuracy of the classifiers.

As it is pictured in figure 2 at the end of the paper, there are three main important aspects in IENB algorithm:

– **Calculation of confidence intervals**

Given the dataset, the first step is to calculate the confidence intervals for each conditional probability and for each class probability. For the calculation of the intervals it must be computed first the point estimations of these same parameters (see section 3.1).

In this way, each conditional probability that has to be estimated from the dataset $\hat{p}_{k,r}^i = \hat{P}(X_k = x_k^r | C = c^i)$ must be computed with the next confidence interval, as it is introduced in section 3.1. The contribution of this paper is addressed towards this approach.

For $k = 1, \dots, n; i = 1, \dots, r_0; r = 1, \dots, r_k$

$$\left(\hat{p}_{k,r}^i - z_\alpha \sqrt{\frac{\hat{p}_{k,r}^i(1 - \hat{p}_{k,r}^i)}{N_i}}; \hat{p}_{k,r}^i + z_\alpha \sqrt{\frac{\hat{p}_{k,r}^i(1 - \hat{p}_{k,r}^i)}{N_i}} \right) \quad (5)$$

where,

r are the possible values of variable X_k

i are the possible values of the class

$\hat{p}_{k,r}^i$ is the point estimation of the conditional probability $P(X_k = x_k^r | C = c^i)$

z_α is the $(1 - \frac{\alpha}{2})$ percentil in the $\mathcal{N}(0,1)$

N_i denotes the number of instances where $C = c^i$

Also, in a similar way, the probabilities for the class values $\hat{p}_i = \hat{P}(C = c^i)$ are estimated with the next confidence interval,

$$\left(\hat{p}_i - z_\alpha \sqrt{\frac{\hat{p}_i(1 - \hat{p}_i)}{N}}; \hat{p}_i + z_\alpha \sqrt{\frac{\hat{p}_i(1 - \hat{p}_i)}{N}} \right) \quad (6)$$

where,

$\hat{p}_i$ is the point estimation of the probability $P(C = c_i)$

z_α is the $(1 - \frac{\alpha}{2})$ percentil in the $\mathcal{N}(0,1)$

N is the number of cases in dataset

– **Search space definition**

Once the confidence intervals are estimated from the dataset, it is possible to generate as many naïve Bayes classifiers as we want. The parameters of these naïve Bayes classifiers must only be taken inside theirs corresponding confidence interval.

In this way, each naïve Bayes classifier is going to be represented with the next tuple of dimension $r_0(1 + \sum_{i=1}^n r_i)$

$$(p_1^*, \dots, p_{r_0}^*, p_{1,1}^{*1}, \dots, p_{1,1}^{*r_0}, \dots, p_{1,r_1}^{*r_0}, \dots, p_{n,r_n}^{*r_0}) \quad (7)$$

where,
 p^* are the selected values inside each confidence interval.

Thus, the search space for the heuristic optimization algorithm is composed of all the valid tuplas. A tupla is valid when it represents a valid naïve Bayes classifier. Formally,

$$\sum_{i=1}^{r_0} p_i^* = 1; \forall k \forall i \sum_{r=1}^{r_k} p_{k,r}^{*i} = 1 \quad (8)$$

Finally, each generated individual must be evaluated with a fitness function. This fitness function it is based on the percentage of successful predictions on each dataset, what means that we are carrying out one wrapper approach.

– Heuristic search for the best individual

Once the individuals and the search space are defined, one heuristic optimization algorithm is ran in order to find the best individual.

To deal with the heuristic search in this work, EDAs –estimation of distribution algorithms– algorithms have been selected. EDAs [16, 15] are non-deterministic, stochastic and heuristic search strategies that belong to the evolutionary computation approaches. In EDAs, the number of solutions or individuals are created every generation, evolving once and again until a satisfactory solution is achieved. In brief, the characteristic that most differentiates EDAs from other evolutionary search strategies such as GAs is that the evolution from a generation to the next one is done by estimating the probability distribution of the fittest individuals, and afterwards by sampling the induced model. This avoids the use of crossing or mutation operators, and, therefore, the number of parameters that EDAs requires is reduced considerably.

5 Experimentation

5.1 Datasets

Results are compared for 21 classical datasets –see table 1–, also used by other authors [5]. All the datasets belong to the UCI repository [17], with the exception of *m-on-f-3-7-10* and *corral*. These two artificial data sets were designed to evaluate methods for feature subset selection [9].

5.2 Experimental methodology

To estimate the prediction accuracy for each classifier our own implementation of a naïve Bayes classifier has been programmed. This implementation uses the Laplace correction for the point estimation of the conditional probabilities [7, 9] and deals with missing values as recommended in [1].

However, our new algorithm does not handle continuous attributes. Thus, a discretization step with the method recommended in [2] has been performed using MLC++

Table 1. Description of the data sets used in the experiments

Name	Attributes			Classes	Instances	
	Total	Continuous	Nominal		Learning	Validation
<i>breast</i>	10	10	-	2	699	-
<i>chess</i>	36	-	36	2	3196	-
<i>cleve</i>	13	6	7	2	303	-
<i>corral</i>	6	-	6	2	128	-
<i>crx</i>	15	6	9	2	692	-
<i>flare</i>	10	2	8	2	1066	-
<i>german</i>	20	7	13	2	1000	-
<i>glass</i>	9	9	-	7	214	-
<i>glass2</i>	9	9	-	2	163	-
<i>hepatitis</i>	19	6	13	2	155	-
<i>iris</i>	4	4	-	3	150	-
<i>lymphography</i>	18	3	15	4	148	-
<i>monf-3-7-10</i>	10	-	10	2	300	1024
<i>pima</i>	8	8	-	2	768	-
<i>satimage</i>	36	36	-	6	6435	-
<i>segment</i>	19	19	-	7	2310	-
<i>shuttle-small</i>	9	9	-	7	5800	-
<i>soybean-large</i>	35	-	35	19	683	-
<i>vehicle</i>	18	18	-	4	846	-
<i>vote</i>	16	-	16	2	435	-
<i>waveform-21</i>	21	21	-	3	300	4700

tools [11]. This discretization method is described by Ting in [20] that is a global variant of the method of Fayyad and Irani [4].

Nineteen of the datasets has no division between training and testing sets. On these datasets the results are obtained by a *leave-one-out* method inside of the heuristic optimization loop.

On the other hand, two out these twenty one datasets include separated training and testing sets. For these cases, the heuristic optimization algorithm uses only the training set to tune the classifier. A *leave-one-out* validation is performed internally inside of the optimization loop, in order to find the best classifier. Once the best candidate is selected, it is validated using the testing set. The overall process is shown on figure 1.

5.3 Results

Experiments were run in a Athlon 1700+ with 256MB of RAM memory. The parameters used to run EDAs were: population size 500 individuals, selected individuals for learning 500, new individuals on each generation 1000, learning type UMDA [16] and elitism. Experiments were run 10 times with two different percentiles: percentile 0.95 ($z_\alpha = 1.96$) and percentile 0.98 ($z_\alpha = 3.08$).

Results for the UCI problems are shown in the table 2. Results are splitted in two sections, one for the 0.95 percentile ($z_\alpha = 1.96$) and the other for the 0.98 percentile

Fig. 1. Validacin de los clasificadores cuando el conjunto de datos est dividido en conjunto de aprendizaje y conjunto de testeo.

($z_\alpha = 3.08$). Columns in the table are: first, the best value obtained with EDAs, second, the average value of the ten executions, third, the average time –in minutes– for the executions and fourth, the improvement respect to a simple naïve Bayes.

Results are really interesting. Respect to naïve Bayes, using the 0.95 percentile, we obtained an average improvement of 3.57%, and using the 0.98 percentile the improvement scales up to 4.65%. It is possible to extract other conclusions from the results. We can appreciate a clear tendency to improve using the 0.98 percentile. Besides, although this is not always true, better improvements are obtained in the data sets with less number of cases.

6 Conclusion and further work

In this work we have presented a new semi naïve Bayes approach. In our experiments this approach has an average improvement of 4.65% respect to the simple naïve Bayes.

As this is the first time we use this approach, many issues remain for future research. For instance, interval estimation naïve Bayes can also be used with continuous variables. It is possible to estimate intervals for the parameters μ and σ of the gaussian distribution.

Also, it is possible to change the objective of the heuristic search. We can try to maximize the area under the ROC curve and not the percentage of successful predictions. Another viable idea is to combine interval estimation naïve bayes with a feature subset selection. On a first phase it is possible to make a subset selection, and on a second phase to apply interval estimation to the previous results.

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Table 2. Experiment results for Interval Estimation naïve Bayes

<i>Interval Estimation naïve Bayes</i>								
	<i>Percentile 0.95</i>				<i>Percentile 0.98</i>			
	<i>Max</i>	<i>Aver</i>	<i>Time</i>	<i>Improve</i>	<i>Max</i>	<i>Aver</i>	<i>Time</i>	<i>Improve</i>
<i>australian</i>	88.99	88.76	10.4	3.77	90.15	89.91	11.9	4.93
<i>breast</i>	97.71	97.71	1	0.57	97.71	97.71	1	0.57
<i>chess</i>	92.02	91.85	107	8.19	93.74	93.70	119.4	9.91
<i>cleve</i>	86.14	85.91	2.5	2.31	86.47	86.47	1	2.64
<i>corral</i>	92.18	92.03	1	7.81	93.75	93.75	1	9.38
<i>crx</i>	89.28	89.10	13.7	2.90	90.15	89.97	10.3	3.77
<i>diabetes</i>	79.82	79.51	1	2.09	80.08	79.95	1.3	2.35
<i>flare</i>	82.18	82.18	1	1.32	82.65	82.48	2.5	1.79
<i>german</i>	80.80	80.70	23	5.40	81.80	81.60	41.2	6.40
<i>glass</i>	79.91	79.49	1	5.14	81.31	81.31	1	6.54
<i>glass2</i>	88.34	88.34	1	6.13	88.34	88.34	1	6.13
<i>heart</i>	85.93	85.89	1	2.23	86.30	86.30	1	2.60
<i>hepatitis</i>	90.97	90.97	1	5.81	90.97	90.97	1	5.81
<i>iris</i>	96.00	95.93	1	1.33	96.00	96.00	1	1.33
<i>letter</i>	75.08	74.97	301	1.04	75.60	75.37	340	1.56
<i>lymphography</i>	92.57	92.09	1	7.43	95.27	94.66	2.6	10.13
<i>monf-3-7-10</i>	96.45	95.85	2	12.24	100.00	100.00	2	15.79
<i>pima</i>	79.69	79.47	1	1.96	80.08	79.90	1	2.35
<i>satimage</i>	83.33	83.23	221.1	0.87	83.42	83.42	228.4	0.96
<i>segment</i>	93.55	93.53	131.1	1.60	95.45	95.41	155.4	3.50
<i>shuttle-small</i>	99.43	99.43	20.6	0.07	99.90	99.88	22.5	0.54
<i>soybean-large</i>	94.00	94.00	180.9	0.88	94.58	94.58	188.5	1.46
<i>vehicle</i>	66.78	66.56	19.2	5.31	70.33	70.20	26.5	8.86
<i>vote</i>	91.49	91.47	1	1.38	95.17	95.17	2	5.06
<i>waveform-21</i>	83.08	83.03	133.8	1.44	83.60	83.52	162.3	1.96

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Fig. 2. Interval Estimation naïve Bayes.